NATURAL DISPERSION IN A SMALL LAKE

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Abstract—Videotapes of dye dispersion experiments in the epilimnion of a small lake were successfully analyzed with image-processing techniques. The rate of horizontal dispersion from dye releases in the center of the lake was consistent with previous observations in oceans and large lakes. We derive a power-law relationship that relates the apparent diffusivity to the characteristic length scale of the dye patch. This relationship applies to all three data sets and is valid over length scales ranging from 10 m to more than 100 km. Dye releases from a point on the shore of the same lake dispersed slightly faster than the central releases. This observation, while clearly not universal, has prompted the successful shore-based fertilization of several small lakes.

We used video image-processing techniques to study the horizontal dispersion of dye in the epilimnion of a small lake. Our study was motivated by the need to evaluate the potential effectiveness of point-source fertilizer applications into small lakes. Similar studies, notably those of (Okubo 1971) in the surface layer of the ocean and of (Murthy 1976) in the epilimnion of Lake Ontario, have revealed empirical relationships between dispersion rates and the size of the dye patch. Even though these empirical relationships do not explicitly account for the various mechanisms causing dispersion (including turbulent diffusion, shear dispersion, differential heating and cooling, Langmuir circulation, and frontogenesis, see Imberger and Patterson 1990), the results are of considerable practical usefulness in providing preliminary estimates of dispersion rates. However, empirical results obtained in large lakes and in the ocean are not necessarily appropriate for small lakes, hence the need for the present study.

From the perspective of fertilization we are primarily concerned with horizontal dispersion in the epilimnion. The disparity between horizontal and vertical length scales in the epilimnion allows us to assume that vertical mixing occurs relatively quickly. We are not concerned with hypolimnetic mixing because the fertilization of epilimnetic waters is most effective in stimulating primary production, and, theoretically, the yield of fish. We examined dye releases from the lakeshore as well as the lake center to demonstrate the potential effectiveness of shore-based addition of liquid fertilizer.

Our dye dispersion experiments were performed in Twin West Lake (49°21'N, 122°15'W), which is ~75 km east of Vancouver at an elevation of 500 m in the Coastal and Insular Mountain limnological region of British Columbia. This lake is approximately bowl-shaped (Northcote and Larkin 1956). It is typical of many small oligotrophic lakes being considered for fertilization in an attempt to increase the yield of salmonids for recreational angling. Twin West Lake is oligotrophic, with summer chlorophyll a concentrations <2.0 µg liter⁻¹ and Secchi depths ~7 m (B.C. Min. Environ. Fish. Proj. Rep. RD 19). A typical summer density stratification consists of an epilimnion with temperatures in excess of 20°C separated by a sharp thermocline from a hypolimnion with temperatures dropping to 4°C. The thermocline is relatively close to the surface (~3 m) because the lake is well sheltered from prevailing winds.

The motions of dye released at the shore and lake center were photographed and videotaped from a vantage point as shown in Fig. 1. Experiments were performed on 2, 18, and 24 August 1988. On each occasion, central releases of sodium fluorescein (green) dye and shoreline releases of Rhodamine WT (red) dye were made. Typically 2 liters of 20% solutions of dye were slowly poured into the lake. Initially the solutions were slightly more dense than the lake water, but as they sank they entrained sufficient quantities of water so that the thermocline acted as a barrier to the dye patch. The resulting columns of dye were of relatively uniform concentration from the surface to the thermocline.

The subsequent dispersion of each dye patch was recorded on videotape and 35-mm film. Most of the lake could be seen from the vantage point with a 135-mm lens. A variety of film types and filters were tested; the best results were obtained with 25 ASA film and a combination of polarizing and yellow filters. A Super VHS video camera was connected to an external 12-V automotive battery and recorded for 3 s every 2 min. The visual observations were supplemented by measurements of windspeed and direction taken at 2-min intervals from a data logger anchored at the lake center.

Photographs of the experiment performed on 24 August 1988 (Fig. 2) show the dispersion of dye from both a central release and a shore release. The central release responded directly to wind-driven currents. At the surface it moved to the right at an average speed of 2.8 cm s⁻¹, whereas in the lower portion of the epilimnion it was advected to the left by a return current of ~1 cm s⁻¹. Given that the windspeed was ~1 m s⁻¹ throughout the experiment, these observations are in accord with those of Baines and Knapp (1965) and Wu (1975). Small changes in wind direction between 1310 and 1336 hours contributed to horizontal dispersion by shearing the dye near the surface with respect to the dye at depth (Fig. 2c, d). Thus the horizontal dispersion of the dye cloud is caused by a combination of the vertical velocity gradient and changes in the wind direction as well as by turbulent mixing.

The motion of the shoreline release is more complicated. Initially, wind-driven currents transport the dye along the shoreline (Fig. 2a). However, logs that have fallen into the lake redirect some of the dye toward the center of the lake (Fig. 2c). Eventually the dye cloud becomes quite distorted (Fig. 2d), presumably because currents at the side of the lake are affected by the irregular bottom topography. In addition, in a process known as
differential heating (Imberger and Patterson 1990), the reduced depth around the perimeter of the lake may lead to higher temperatures, causing buoyant surface flows toward the center of the lake.

In processing the videotape of the experiments, we followed a procedure consistent with that prescribed by Okubo (1971) which did not rely on any subjective decisions on the part of the analyst. Our first step was to transfer the video image to a Macintosh II PC using a Data Translation frame-grabber board (QuickCapture) and then to a SUN workstation as a TIFF image file. Each video image was represented by matrix of 640×480 light intensities measured on a gray scale from 0 to 255. The oblique video images of the lake were geometrically transformed to give a bird's-eye view of the lake surface. This transformation requires that the map coordinates of four points visible in the video images are known (the details of this transformation are given by Moffitt and Mikhail 1980). Fourteen marker buoys, some of which are visible in Fig. 2, were anchored in the lake on a 60-m grid to

Fig. 1. Schematic of the experimental setup (not to scale).

Fig. 2. Dispersion of a central release of sodium fluorescein (green) dye and a shore release of Rhodamine WT (red) dye into Twin West Lake on 24 August 1988. The central release was at 1248 and the shore release at 1259 hours. The marker buoys visible on the lake's surface were anchored on a 60-m grid. The photographs were taken at (a) 1310, (b) 1318, (c) 1325, and (d) 1336 hours. The thin orange line stretching across the lake is a 6-mm-thick nylon rope that does not play a part in the present study.
ensure that at least four buoys were visible in the video image at all times. The position of these buoys was determined to within 50 cm by triangulating from control points established around the perimeter of the lake.

Variations in background light intensity are visible in Fig. 2. Typically, there were small-scale random fluctuations in intensity imposed on an approximately linear variation in background intensity, due to the variations in the intensity of reflected light from the lake’s surface with distance from the observation point. The contours of light intensity associated with the central release are shown in Fig. 3a. These contours were obtained by removing the linear component of the variation in background intensity from the bird’s-eye view image. Next we followed Okubo (1971) and assigned each curve of constant intensity within a dye patch with an equivalent radius, \( r_e \), such that the area enclosed by the isoconcentration curve was the same as that of a circle of radius \( r_e \). Both the dye itself and the random variations in light intensity contributed to the resulting radially symmetric distribution of intensity (upper curve, Fig. 3b). The random component was identified by following the above procedure while ignoring that part of the image containing the dye. After subtracting this random component we obtained the radially symmetric distribution of intensity due to the dye \( i(t, r_e) \) (lower curve, Fig. 3b), where \( t \) is the time since the release of the dye.

In the laboratory the intensity of fluorescence is proportional to the dye concentration (Weidemann 1972). Given that vertical mixing will occur relatively quickly, we assume the intensity of the dye that we observe is proportional to the vertically integrated concentration—the quantity of interest. Some errors are introduced by the attenuation of light through the epilimnion (see Ichiy and Plutchak 1966); however these are small because the epilimnion is so thin and the lake water so clear.

Eventually the dye could no longer be distinguished effectively from the background. This occurred between 22 and 46 min after the dye release (we were not concerned about photochemical decay of the dyes, because it only becomes significant after several hours for sodium fluorescein, and after many days for Rhodamine WT, see Smart and Laidlaw 1977). At the other extreme we followed Okubo’s (1971) recommendation to ignore data taken before the dye patch had grown large enough to have “forgotten” its initial conditions (i.e. at least 10 times its initial size). Given that the initial length scale of our dye patches was ~1 m, we only used data after the length scale had reached 10 m. Similarly, because the release of dye took up to 1 min, we only used data obtained at least 10 min after the release.

After following the data reduction procedure outlined above, we obtained 8, 10, and 19 data points for the central releases on 2, 18, and 24 August and 6, 0, and 10 data points for the shore releases. No data were obtained for the shore release of 18 August because we could not effectively distinguish the dye patch from the background. We followed Okubo’s (1971) analysis and calculated the variance of the concentration distribution about its center of mass:

\[
\sigma_r^2 = \frac{\int_0^{r_e} i(t, r_e) 2\pi r_e \, dr_e}{\int_0^{r_e} i(t, r_e) 2\pi r_e \, dr_e}.
\] (1)

Ideally we would have defined diffusivity in terms of the
time rate of change of an ensemble average of this variance over a large number of releases. Unfortunately, the difficulties involved in obtaining field data are such that this approach is not feasible. To provide useful information from a limited data set, Okubo (1971) defined an apparent diffusivity:

\[ K_a = \frac{\sigma_r^2}{4t} \]  

(2)

and arbitrarily defined the characteristic length scale or “size” of the dye patch:

\[ \ell = 3 \sigma_r. \]

(3)

If the radially symmetric distribution is Gaussian, i.e.

\[ i(t, r) = i(t, r_c) \exp\left[ -\left( r / \sigma_r \right)^2 \right], \]

then 90% of the dye remains within a diameter of \( 2[\ln(10)]^{-1/2} \sigma_r \approx 3 \sigma_r \), or alternatively within the area,

\[ A_{90} = \pi \ln(10) \sigma_r^2 = 7.23 \sigma_r^2. \]  

(4)

The variation of \( \sigma, \ell, \) and \( K_a \) with \( t \) are presented in Table 1 for each of the dye releases. Although the variation of \( \ell \) with \( t \) (Fig. 4) is similar for each release, it is worth noting that the shore releases dispersed more rapidly than the central releases.

We have followed the procedures set down by Okubo (1971) to allow direct comparison of results. Although there have been a number of studies of horizontal dispersion in the surface layer, we can make only limited...
Fig. 4. Variation of the characteristic length scale, $\ell$, with time for all the Twin West Lake dye release experiments. Dashed line is the line of best fit for the central release (i.e. $\ell = 9.4 \times 10^{-3}$ m$^{1.07}$). Note that the length scale is larger for the shoreline releases.

use of most of them, because they have not presented the results in a form that allows direct comparison with Okubo (1971). For example, in his discussion of List et al. (1990), Okubo (1992) showed that subtle differences in the method used to calculate dispersion coefficients and length scales can preclude direct comparison. To our knowledge the only other extensive dye study of horizontal dispersion in the surface layer that is fully compatible with Okubo (1971) is that of Murthy (1976). We will compare our central release data with these two studies. The shore release data will not be compared because Okubo (1971) only considered dye patches that were not constrained by any shoreline.

Initially the rate of growth of the central releases was primarily dependent on windspeed. However, once the patches were sufficiently elongated, their growth became sensitive to changes in the wind’s direction. The dispersion was greatest on 2 and 24 August because changes in wind direction caused the dye patches to shear at an angle to their major axes (see Fig. 2). This did not occur on 18 August.

Our results, as well as those of Okubo (1971) and Murthy (1976), can be expressed as

$$K_a = \alpha \ell^\beta,$$

where the parameters $\alpha$ and $\beta$ incorporate the effects of many factors (e.g. various meteorological factors including surface heat fluxes, and the windspeed, direction, and fetch; various oceanographic factors including the vertical density profile, the background current field, and the turbulence levels; and the inevitable measurement errors).

The diffusion diagram for all three data sets (Fig. 5), reveals the consistency of the Twin West Lake data set with the Okubo-Murthy data set, as do the values of $\alpha$, $\beta$, and SE($\beta$) listed in Table 2. The line of best fit to the complete data set yields $\beta = 1.10$ and $\alpha = 3.2 \times 10^{-4}$ m$^{0.9}$s$^{-1}$. The 95% confidence limits plotted on Fig. 5 show that the diffusivity can vary by a factor of $\sim 2.5$ on either side of the predicted values. This degree of uncertainty is to be expected given the myriad other factors that can influence diffusivity besides the length scale of the dye patch, and given that an extremely wide range of length scales is covered by the data. The exponent $\beta$ is close to 1.1 in each case except for Murthy’s (1976) data set with $\beta = 1.35$. We do not attach any particular significance to this difference, because Murthy’s (1976) data set contains relatively few points (11), all of which plot within, or close to, the 95% confidence limits of the line of best fit for the complete data set (see Fig. 5). With $\alpha = 3.2 \times 10^{-4}$ m$^{0.9}$s$^{-1}$ and $\beta = 1.10$ we can rearrange Eq. 2–5 to obtain

$$\sigma_{eff}^2 = 5.6 \times 10^{-2} \ell^2,$$

$$\ell = 7.1 \times 10^{-1}t^{1.1},$$

and

$$A = 4.1 \times 10^{-5}t^{2.22}$$

where $\ell$ is measured in m, $\sigma_{eff}^2$ and $A_{eff}$ in m$^2$, and $t$ in s.

There is theoretical support for the celebrated $4/3$ power law, commonly written as

$$K_a = c \ell^{4/3}$$

where $c$ is the energy dissipation rate, and $\ell$ is a constant of order unity. This law was proposed as a model of atmospheric diffusion by Richardson (1926) and for oceanic diffusion by Stommel (1949). Using the hypotheses of Kolmogorov (1941), Batchelor (1950) showed that under certain conditions the similarity theory for homoge-
Table 2. Relationship between the apparent diffusivity, $K_a$, and the length scale, $\ell$, for the data sets of Okubo (1971), Murthy (1976), and our central releases at Twin West Lake, and combinations of these data sets. Regression analysis was carried out for the equation $K_a = \alpha \ell^\beta$. The number of data points in each data set $N$, the parameter $\alpha$, the exponent $\beta$, its standard error $SE(\beta)$, and the correlation coefficient $r$ are given.

<table>
<thead>
<tr>
<th>Data set</th>
<th>$N$</th>
<th>$\alpha$ (m$^2$ s$^{-1}$)</th>
<th>$\beta$</th>
<th>SE(\beta)</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Okubo (O)</td>
<td>57</td>
<td>$2.8 \times 10^{-4}$</td>
<td>1.10*</td>
<td>0.04</td>
<td>0.97</td>
</tr>
<tr>
<td>Murthy (M)</td>
<td>11</td>
<td>$6.6 \times 10^{-3}$</td>
<td>1.35</td>
<td>0.06</td>
<td>0.99</td>
</tr>
<tr>
<td>Twin West, central (TW)</td>
<td>37</td>
<td>$3.6 \times 10^{-4}$</td>
<td>1.07*</td>
<td>0.06</td>
<td>0.95</td>
</tr>
<tr>
<td>O + M</td>
<td>68</td>
<td>$2.5 \times 10^{-4}$</td>
<td>1.13</td>
<td>0.04</td>
<td>0.96</td>
</tr>
<tr>
<td>O + M + TW</td>
<td>105</td>
<td>$3.2 \times 10^{-4}$</td>
<td>1.10</td>
<td>0.02</td>
<td>0.99</td>
</tr>
</tbody>
</table>

* Okubo (1971) obtained $\beta = 1.15$ using a straight line visually fit to his data; however, a least-squares best fit gives $\beta = 1.10$.

Homogeneous, isotropic turbulence is not found in nature, particularly in the small lake that we studied, and experimental studies have generally yielded $\beta < 4/3$. For example Kullenberg (1972) obtained $\beta = 1.31$ for dispersion in stratified subsurface waters, Randerson (1972) obtained $\beta = 1.17$ for dispersion in an expanding nuclear debris cloud, and Borthwick (1980) obtained $\beta = 1.12$ and 1.08 at depths of 1 and 5 m in Swansea Bay.

Ozmidov (1965), Okubo and Ozmidov (1970), and Okubo (1974) have attempted to reconcile the 4/3 law with the empirical data by noting that energy is fed into ocean turbulence at a range of scales; by wind waves at order 10 m, by inertial and tidal motions at order 10 km, and by atmospheric pressure systems at order 1,000 km. They argue that since the dissipation rate is actually the energy pass-through rate in the energy cascade from large energy-containing eddies to small dissipative eddies, it will increase with a decrease in scale, giving a better fit with the 4/3 law.

An alternative explanation for $\beta < 4/3$ is that even in homogeneous, isotropic turbulence, energy dissipation does not occur uniformly. There are patches of intense small eddies where dissipation is much higher than in the surrounding fluid. This intermittency results in a reduction of $\beta$, the magnitude of which is not precisely known. Competing theories suggest a reduction of up to $\sim 0.17$, Frisch et al. (1978). A correction of 0.17 would account for most of the difference between the 4/3 law and experimental results.

The above arguments show that purely turbulent diffusion might explain the observed behavior, but we certainly should not preclude other possible mechanisms. In our study it appears that horizontal dispersion is produced by the combination of a vertical velocity gradient and turbulent mixing in the same direction in a process known as "shear dispersion." Consideration of shear dispersion can yield several possible relationships (see Kullenberg 1972). It is worth noting that for the case of a uniform velocity gradient and a constant vertical diffusivity, the longitudinal variance, $\sigma_x^2 \propto \rho$ (providing $t$ is sufficiently large), and the transverse variance, $\sigma_y^2 \propto t$. If the distribution of concentration in each direction is Gaussian, then $\sigma_x^2 = 2 \sigma_y \sigma_y \propto t^2$, and from Eq. 1-3 $K_a \propto t \propto \ell$, yielding $\beta = 1$. This is the same value of $\beta$ as obtained by Joseph and Sendner (1958) who hypothesized that diffusivity is proportional to the product of a "diffusion velocity" and the length scale.

It is beyond the scope of the present study to investigate all the competing mechanisms that may in combination explain the observed results. Further studies, such as those of Kullenberg (1972) and List et al. (1990), relating dispersion to its underlying causes are needed.

We have already noted that for a given diffusion time the characteristic length $\ell$ was greater for the shore releases than for the central releases. Although this result is based on very limited data, it indicates that under critical (light wind) conditions a shoreline release of fertilizer can be as effective as, if not more effective than, a central release. Of course the dispersion from a shore release is sensitive to the location of the release (it would be unwise to expect significant dispersion from a semi-enclosed bay). In addition, nutrient uptake by the periphyton-macrophyte-benthic community in the nearshore area must be considered. Nevertheless, it is worth noting that although Rhodamine WT concentrations the day after the release were uniform throughout the epilimnion, there were no detectable concentrations in the hypolimnion. Thus, in this case, fertilizer application from the shore of the lake would have been successful given the appropriate biological conditions. These findings prompted the successful shore-based fertilization of a larger (78 ha) lake (3 km ENE of Twin West lake) in summer 1989 (B.C. Min. Environ. Fish. Proj. Rep. RD 19) and several additional lakes (9.2–63.3 ha) on Vancouver Island in summer 1993.

Compared with the alternative of crossing the dye patch with a boat to obtain in situ measurements of concentration, our method of videotaping the dye patch has the advantages and disadvantages common to most remote-sensing techniques. The main disadvantage is that we are not measuring concentration directly, so both fluctuations in background intensity and the attenuation of light within the water column introduce errors. Fortunately, these errors are small and of minor importance because we are seeking measures of patch size rather than
absolute measurements of concentration. Our method can only be used in daylight and while dye concentrations are high enough to be visible. A suitable vantage point must also be available. The main advantages of our method are that it is nonintrusive, inexpensive, and provides an instantaneous and complete picture of the dye patch. The improvement in temporal resolution is dramatic because it takes a considerable period of time for a boat to sample a dye patch in sufficient detail to obtain tolerable contours of dye concentration at a given depth. In addition, the motion of the boat may alter the dye distribution. Furthermore, our method gives estimates of the vertically integrated concentration through the epilimnion, which is the quantity of primary interest.

We would have liked to have compared our results against in situ measurements; unfortunately we did not have the resources to do this effectively. However, we have made an indirect, partial verification of our results. Three independent observers traced the outline of the dye patch and obtained estimates of its area that bracketed the values obtained by image processing. On average, \( A_0 \) was \( \approx 10\% \) less than the visual estimates.

In summary video image processing was effectively used to quantify the dispersion of dye in the epilimnion of a small lake. The rate of spreading of dye released from the center of the lake was consistent with that observed in oceanographic diffusion experiments (Okubo 1971) and in Lake Ontario (Murthy 1976). The variation of apparent diffusivity, \( K_\alpha \), with the characteristic length scale of the patch, \( \ell \), for the combination of all three data sets is approximated by

\[
K_\alpha = 3.2 \times 10^{-4} \ell^{1.1}.
\]

Values obtained with this equation have a 95% probability of being accurate to within a factor of \( \sim 2.5 \) over length scales ranging from 10 m to \( > 100 \) km. The importance of shear dispersion rather than purely turbulent diffusion in our experiments is one of several possible reasons that the exponent in Eq. 8 is significantly different from the commonly used value of 4/3.

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