This short note defines and illustrates three terms that are important in dealing with approximate numbers.

Every approximate number has a specific number of significant digits (or significant figures). These are digits in the number which convey actual numerical information, and are not just written down to show us where the decimal point is located. Thus

(i.) all nonzero digits are significant.

(ii.) all zeros which are between nonzero digits are significant.

(iii.) all zeros to the right of the decimal point are significant if they follow nonzero digits in the number.

(iv.) zeros which are present only to show the position of the decimal point are not significant.

(v.) zeros which can be omitted without affecting the numerical value are not significant. (This rule overlaps rule (iv.), but includes so-called "leading zeros" sometimes written in the whole number parts of decimal values.)

The first rule covers most situations. The tricky cases are situations with digits which are zeros, because the digit zero has two roles in decimal numbers. One role is to indicate the value zero at a certain position (as in 105 equals one hundred plus zero tens plus five ones). The other role is to tell us where the decimal point should be located (as in a number like 0.0035).

**Examples:**

142.56 has five significant digits because all five digits in the number are nonzero (rule (i.) above).

3001.378 has seven significant digits. It has seven digits and all seven are significant. Five of the seven digits are nonzero digits, and so are significant by rule (i.). The two zeros are between significant digits, and so are themselves significant by rule (ii.). You can think of these two digits as being as significant as the other digits in the number because they indicate the value here, for instance, has no hundreds (as opposed to one hundred, or two hundreds, etc.) and that it also has no tens (as opposed to one ten, or two tens, etc.)

5.40 has three digits and all three are significant. The two nonzero digits are significant by rule (i.). The zero at the end is also significant by rule (iii.) – it is to the right of the decimal point and follows nonzero digits in the number. This zero indicates that the number has been measured to two decimal places (or, its uncertainty of measurement is ±0.005, following the ideas described in the previous document). If the zero in this number was not the result of measurement (that the value has 5 ones and 4 tenths, and zero hundredths, as opposed to one hundredth or two hundredths, etc), then it should not have been written.
In each of the above examples, all digits in each of the numbers were significant. The only troublesome digits are zeros, and in the examples above, the zeros that did appear were significant due to rules (ii.) and (iii.). They are significant because they indicate that the digit in that position has been measured or observed and is known to be zero rather than 1 or 2 or 3 or 4 or 5 or 6 or 7 or 8 or 9. The complexity of the formal rules given at the beginning is an attempt to distinguish between zeros which are really the result of measurement and zeros which are a consequence of our style of number writing.

Now we’ll give some examples of numbers containing digits which are not significant.

**Example:**

00352.6 has four significant digits. Obviously the two zero digits on the left convey no information – it’s not clear why they were even written down here. This is an example of rule (v.).

0.00516 has three significant digits. The three zero digits here are intended simply to show the location of the decimal point. This is an example of rule (iv.).

0.05016 has four significant digits. The zero between the ‘5’ and the ‘1’ is significant, by rule (ii.). The two zeros on the left are not significant by rule (iv.).

You might argue that all three zeros should be considered significant because just as the rightmost zero (between the ‘5’ and the ‘1’) indicates no thousandths, so, the leftmost zero indicates no ones, and the second zero indicates no tenths. But this argument misses an important point. Suppose this value was the length of an object measured in metres:

\[ \text{length of this object} = 0.05016 \text{ m} \]

If we simply re-expressed this length in millimetres (we would need to multiply the number of metres by 1000 to get the equivalent length expressed in millimetres), we could then write

\[ \text{length of this object} = 5.016 \text{ mm} \]

This is still the same physical measurement of the same physical length but now those two zero digits on the left are not required. Therefore they could not have been significant. This sort of observation is the source of rules (iv.) and (v.), indicating when zero digits are not significant.

0.050160 has five significant digits. The two zeros on the left are not significant by rule (iv.), as discussed in the previous example. The third zero is significant by rule (ii.) – it is between two other significant digits. The fourth zero is significant by rule (iii.) – it is to the right of the decimal point and to the right of a significant digit. The idea is that if this rightmost zero was not significant, it would not have been written down at all.

This just leaves one problematic type of situation – numbers with no explicit decimal point but with trailing zeros. Examples are

20, 43000, 50700
The problem is, the rightmost zeros may be significant because they are the result of actual measurement and so convey actual information, or they may be present simply to locate the (assumed) decimal point. There is not good solution to the interpretation of this way of writing numbers that is recognized by all practitioners. However, the best rule seems to be to consider that all such trailing zeros are not significant, and to resort to scientific notation (see a later note in this series) to write the number if you wish to indicate that one or more of such trailing zeros in whole numbers are significant. We’ll give examples of how to do this when we describe scientific notation.

After that lengthy description of significant digits, the remaining two terms to be defined here are quite easy to understand.

By the **accuracy** of an approximate number, we mean the number of significant digits it has.

By the **precision** of an approximate number, we mean the actual position of the rightmost significant digit. If that digit is to the right of the decimal point, we state the position as so many decimal places. If that position is to the left of the decimal point, it is more common to state the precision using words like “tens,” “hundreds,” “thousands,” etc.

**Examples:**

321.56 has an accuracy of 5 significant digits and a precision of 2 decimal places.

3.2156 has an accuracy of 5 significant digits and a precision of four decimal places.

321560 has an accuracy of 5 significant digits and a precision of tens (assuming the rightmost zero is not significant).

0.00000003 has an accuracy of 1 significant digit and a precision of 8 decimal places.

325,000,000 has an accuracy of 3 significant digits and a precision of millions.

Note that the concepts of precision and accuracy are quite distinct. A number can have high accuracy (many significant digits) but not very high precision. Similarly, a number can be very precise, but not very accurate. This is why we need two different terms here – you cannot say “accuracy” when you mean “precision,” or vice versa.