Properties of Parallel Lines

Two lines are said to be parallel if

(i) they both lie in the same plane, and,
(ii) they do not intersect (or cross each other)

Since a line is considered to extend indefinitely in both directions, this definition really means that parallel lines never cross no matter how far you check in either direction.

A transversal is a third line that crosses a pair of parallel lines on a slant, as shown in the illustration to the right. As the transversal crosses the two parallel lines, eight angles are formed, numbered 1 through 8 in the illustration. You can see that four of the angles are quite large, and four of them are quite small. A very important set of properties of these angles is that the angles which appear to be the same in the illustration really are exactly the same. Thus,

\[ \angle 1 = \angle 3 = \angle 5 = \angle 7 \]

and

\[ \angle 2 = \angle 4 = \angle 6 = \angle 8 \]

Also, pairs of adjacent angles always add up to 180°, as you can easily see from the figure. Thus

\[ \angle 1 + \angle 2 = 180^\circ, \quad \angle 2 + \angle 3 = 180^\circ, \quad \angle 3 + \angle 4 = 180^\circ, \quad \angle 5 + \angle 6 = 180^\circ, \quad \text{etc.} \]

There is some terminology when talking about this situation:

- angles in the same relative position around the two intersection points are called corresponding angles. Thus \( \angle 1 \) and \( \angle 5 \) are corresponding angles, as are \( \angle 4 \) and \( \angle 8 \), \( \angle 2 \) and \( \angle 6 \), and also \( \angle 3 \) and \( \angle 7 \). Corresponding angles are equal.

- \( \angle 3 \) and \( \angle 5 \) are called alternate interior angles. \( \angle 4 \) and \( \angle 6 \) are also alternate interior angles. Alternate interior angles are equal.

- \( \angle 2 \) and \( \angle 8 \) are called alternate exterior angles. \( \angle 1 \) and \( \angle 7 \) are also alternate exterior angles. Alternate exterior angles are equal.

These properties of parallel lines are important in understanding and exploiting the properties of similar triangles – soon to be described in these notes.

These properties also work both ways. For instance, all it takes to determine that two lines are parallel is to demonstrate that when a transversal is drawn across them, one of the pairs of corresponding angles formed is equal.
Example: Determine the values of angles A, B, C, and D, in the figure to the right. Assume that the horizontal lines are parallel.

Solution:

Since angle B is the alternate interior partner of the given angle of $70^\circ$, and alternate interior angles are equal, we have immediately that

$$B = 70^\circ$$

Angle C and the given angle of $70^\circ$ are corresponding angles, and corresponding angles are equal, so

$$C = 70^\circ.$$

Angles A and B are adjacent, so they must add up to $180^\circ$. Thus

$$A + B = A + 70^\circ = 180^\circ$$

so

$$A = 180^\circ - 70^\circ = 110^\circ$$

Similarly, angle D is adjacent to the given angle of $70^\circ$, so the two must add up to $180^\circ$. That is

$$D + 70^\circ = 180^\circ.$$

so

$$D = 180^\circ - 70^\circ = 110^\circ$$

Thus, our solution is:

$$A = 110^\circ, B = 70^\circ, C = 70^\circ \text{ and } D = 110^\circ.$$

While we're on the topic of lines and angles, just a reminder of one more simple, but very useful property. When two lines cross, as shown in the diagram to the right, they form four angles.

- $\angle 1$ and $\angle 3$ are said to be vertical angles (or sometimes vertically opposite angles, though the word “opposite” is a bit redundant)

- $\angle 2$ and $\angle 4$ also form vertical angles.

As is fairly obvious from the diagram, vertical angles are equal. Thus

$$\angle 1 = \angle 3$$

and

$$\angle 2 = \angle 4$$